Continuity

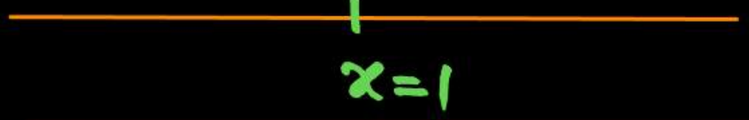
A Function is continuous at x=c iff it is both left as well as right continuous at $x = c$

i.e iff $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$

F(x)

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

L.H.L R.H.L



$\lim_{x \to 1^-} F(x) = \lim_{x \to 1^+} F(x) = F(1)$

DOWNLOAD : GAMMA CAREER CLASSES AAP!

Important Point

1. Modulus functions is Continuous on R.
2. Trigonometric functions are continuous in their respective domains.
3. Exponential function is continuous on R.
4. Every Polynomial functions is continuous on R.
6. Greatest integer function is continuous on all non-integral real number.

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

Discuss the continuity of $f(x)$ given by $f(x) = \begin{cases} 1 & x < 0 \\ x-2 & x \geq 0 \end{cases}$ **at** $x = 0$

L·H·L $\quad F(x) = 1$

R·H·L $\quad F(x) = x-2$

$$\lim_{x \to 0^+} F(x) = \lim_{x \to 0^+} (x-2) = 0-2$$

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

$F(x) = x-2$

$F(0) = 0-2$

$\boxed{F(0) = -2}$

$\boxed{\text{L·H·L} = \text{R·H·L} = F(x)}$

$1 \neq -2 = -2$

$f(x)$ is discon.

DOWNLOAD : GAMMA CAREER CLASSES AAP!

Determine the value of the constant 'k' so that the function

$$f(x)=\begin{cases}\frac{kx}{|x|} & if\ x<0\\ 3 & if\ x\geq 0\end{cases}$$ continuous at $x=0$

R·H·L $\lim_{x\to 0^+} f(x) = 3$

L·H·L $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{kx}{|x|} = \lim_{x\to 0} \frac{kx}{-x} = \lim_{x\to 0} k = k$

$f(x) = 3$

L·H·L = R·H·L = F(x)

$k = 3 = 3$

$k = 3$

GAMMA CAREER CLASSES
MATHS BY SHWARYA
DOWNLOAD : GAMMA CAREER CLASSES AAP!

Let $f(x)=\begin{cases}\frac{\sin x}{x}+\cos x & when\ x\neq 0\\ 2 & when\ x=0\end{cases}$ show that f is continuous at x=0

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{\sin x}{x}+\cos x\right)$$

$$= \lim_{x\to 0}\frac{\sin x}{x} + \lim_{x\to 0}\cos x$$

$$= 1 + \cos 0 = 1+1 = 2$$

$$f(x) = 2$$

$$f(x) = \lim_{x\to 0} f(x)$$

$\therefore$ f(x) is continuous.

$\because \lim_{x\to 0}\frac{\sin x}{x} = 1$

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

If $f(x) = \begin{cases} kx^2 + 5 & x \le 1 \\ 2 & x > 1 \end{cases}$, Find k so that f may be continuous at x=1

L·H·L $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} kx^2 + 5$

$= k(1)^2 + 5$

$\lim_{x \to 1^-} f(x) = k + 5$

$f(x) = kx^2 + 5$

$f(1) = k(1)^2 + 5 = k + 5$

R·H·L $\lim_{x \to 1^+} f(x) = 2$

$kx^2 + 5$ (L·H·L) | R·H·L = 2

$x = 1$

L·H·L = R·H·L = F(x)

$k + 5 = 2 = k + 5$

$k + 5 = 2$

$k = 2 - 5$

$k = -3$

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

Differentiability

A functions f(x) is derivable or differentiable in its domain iff

$\lim_{h\to 0} \frac{f(x-h)-f(x)}{-h} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, and is finite

The value of above limit is denoted by f'(x)and is called the derivative of f(x)

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \log x = \frac{1}{x}$$

$$\frac{d}{dx} a^x = a^x \log a$$

$$\frac{d}{dx} 2 = 0$$

$$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

GAMMA CAREER CLASSES

DOWNLOAD GAMMA CAREER CLASSES AAP!

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\operatorname{cosec}^{-1}x = \frac{-1}{x\sqrt{x^2-1}}$$

GAMMA CAREER CLASSES

DOWNLOAD GAMMA CAREER CLASSES AAP!

Find the derivative of $\sin(\sin x^2)$ at $x = \sqrt{\frac{\pi}{2}}$

Let $y = \sin(\sin x^2)$

$$\frac{d}{dx} y = \frac{d}{dx} \sin(\sin x^2)$$

$$\frac{dy}{dx} = \cos(\sin x^2) \frac{d}{dx} \sin x^2$$

$$= \cos(\sin x^2) \cos x^2 \frac{d}{dx} x^2$$

$$\boxed{\frac{dy}{dx} = \cos(\sin x^2) \cos x^2 \times 2x}$$

at $x = \frac{\sqrt{\pi}}{\sqrt{2}}$

$$\frac{dy}{dx} = \cos\left(\sin\left(\frac{\pi}{2}\right)\right) \cos\frac{\pi}{2} \times 2 \times \frac{\sqrt{\pi}}{\sqrt{2}}$$

$$= \cos(1) \times 0 \times 2 \times \frac{\sqrt{\pi}}{\sqrt{2}}$$

$$= 0$$

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

Differentiate functions w.r.t. x

$$\sqrt{\frac{\sec x + 1}{\sec x - 1}}$$

$$y = \sqrt{\frac{\sec x + 1}{\sec x - 1}}$$

$$= \sqrt{\frac{\frac{1}{\cos x} + 1}{\frac{1}{\cos x} - 1}}$$

$$= \sqrt{\frac{1 + \cos x}{1 - \cos x}}$$

$$\sqrt{\frac{1 + 2\cos^2\frac{x}{2} - 1}{1 - 1 + 2\sin^2\frac{x}{2}}}$$

$$y = \sqrt{\frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}} = \sqrt{\cot^2\frac{x}{2}} = \cot\frac{x}{2}$$

$$y = \cot\frac{x}{2}$$

$$\frac{dy}{dx} = \frac{d}{dx}\cot\frac{x}{2}$$

$$\frac{dy}{dx} = -\operatorname{Cosec}^2\frac{x}{2}\frac{d}{dx}\frac{x}{2}$$

$$= -\operatorname{Cosec}^2\frac{x}{2} \times \left(\frac{1}{2}\right)$$

$$\frac{dy}{dx} = -\frac{1}{2}\operatorname{Cosec}^2\frac{x}{2}$$

$$\cos x = 2\cos^2\frac{x}{2} - 1$$

$$\cos x = 1 - 2\sin^2\frac{x}{2}$$

GAMMA CAREER CLASSES
MADHAV LACHHWANI SIR
DOWNLOAD : GAMMA CAREER CLASSES AAP!

Differentiate functions w.r.t. x

$$f(x) = \sqrt{\sin^{-1}(x^2)}$$

$$F'(x) = \frac{d}{dx}\sqrt{\sin^{-1}(x^2)}$$

$$= \frac{1}{2\sqrt{\sin^{-1}(x^2)}}\frac{d}{dx}\sin^{-1}x^2$$

$$= \frac{1}{2\sqrt{\sin^{-1}x^2}} \times \frac{1}{\sqrt{1-(x^2)^2}}\frac{d}{dx}x^2$$

$$= \frac{1}{2\sqrt{\sin^{-1}x^2}} \times \frac{1}{\sqrt{1-x^4}} \times 2x$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{\sin^{-1}x^2}\sqrt{1-x^4}}$$

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

$$\frac{d}{dx} F(x)\, g(x) = F(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} F(x)$$

$$\frac{d}{dx} \frac{F(x)}{g(x)} = \frac{g(x) \frac{d}{dx} F(x) - F(x) \frac{d}{dx} g(x)}{(g(x))^2}$$

GAMMA CAREER CLASSES

DOWNLOAD GAMMA CAREER CLASSES AAP!

Differentiate functions w.r.t. x

$x^2 \cos^{-1}(1-x)$

$y = x^2 \cos^{-1}(1-x)$

d.w.r. to x

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

$$\frac{dy}{dx} = \frac{d}{dx} x^2 \cos^{-1}(1-x)$$

$$= x^2 \frac{d}{dx} \cos^{-1}(1-x) + \cos^{-1}(1-x) \frac{d}{dx} x^2$$

$$= x^2 \times \frac{-1}{\sqrt{1-(1-x)^2}} \frac{d}{dx}(1-x) + \cos^{-1}(1-x) \times 2x$$

DOWNLOAD : GAMMA CAREER CLASSES AAP!

$$= \frac{-x^2}{\sqrt{1-(1-x)^2}}(0-1) + 2x\cos^{-1}(1-x)$$

$$= \frac{x^2}{\sqrt{1-(1-x)^2}} + 2x\cos^{-1}(1-x)$$

GAMMA CAREER CLASSES

DOWNLOAD GAMMA CAREER CLASSES AAP!

Differentiate functions w.r.t. x : $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

Find $\frac{dy}{dx}$ when $x^2 + xy + y^2 = 100$

$$x^2 + xy + y^2 = 100$$

$$\frac{d}{dx}x^2 + \frac{d}{dx}xy + \frac{d}{dx}y^2 = 100$$

$$2x + x\frac{d}{dx}y + y\frac{d}{dx}x + 2y\frac{dy}{dx} = \frac{d}{dx}100$$

$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x + 2y) = -2x - y$$

$$\boxed{\frac{dy}{dx} = \frac{-2x - y}{(x + 2y)}}$$

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

Find $\frac{dy}{dx}$ when $\tan^{-1}(x^2+y^2)=a$

$$\tan^{-1}(x^2+y^2)=a$$

$$\frac{d}{dx}\tan^{-1}(x^2+y^2)=\frac{d}{dx}a$$

$$\frac{1}{1+(x^2+y^2)^2}\frac{d}{dx}(x^2+y^2)=0$$

$$\frac{1}{1+(x^2+y^2)}\left[2x+2y\frac{dy}{dx}\right]=0$$

H.W.

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

If $\sin xy + \cos xy = 1$ and $\tan xy \neq 1, prove$ that $\frac{dy}{dx} = \frac{-y}{x}$

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

Differentiate functions w.r.t. x

$$\frac{3^x}{2+\sin x}$$

$$y = \frac{3^x}{2+\sin x}$$

$$\frac{dy}{dx} = \frac{(2+\sin x)\frac{d}{dx}3^x - 3^x \frac{d}{dx}(2+\sin x)}{(2+\sin x)^2}$$

$$= \frac{(2+\sin x)\,3^x \log 3 - 3^x(0+\cos x)}{(2+\sin x)^2}$$

Ans

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

Differentiate functions w.r.t. x

$e^{\sin x^2}$

$$y = e^{\sin x^2}$$

$$\frac{dy}{dx} = \frac{d}{dx} e^{\sin x^2}$$

$$\frac{dy}{dx} = e^{\sin x^2} \frac{d}{dx} \sin x^2$$

$$= e^{\sin x^2} \cos x^2 \frac{d}{dx} x^2$$

$$= e^{\sin x^2} \cos x^2 \times 2x$$

$$\boxed{\frac{dy}{dx} = 2x\, e^{\sin x^2} \cos x^2}$$

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

Differentiate functions w.r.t. x

$$2\sqrt{\sec(e^{2x})}$$

H.W

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!

If $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$, prove that $\frac{dy}{dx} = 1 - y^2$

$$\frac{dy}{dx} = \frac{(e^x - e^{-x})\frac{d}{dx}(e^x + e^{-x}) - (e^x + e^{-x})\frac{d}{dx}(e^x - e^{-x})}{(e^x - e^x)^2}$$

$$= \frac{(e^x - e^{-x})(e^x - e^{-x}) - (e^x + e^{-x})(e^x + e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2} = \frac{(e^x - e^{-x})^2}{(e^x - e^{-x})^2} - \frac{(e^x + e^{-x})^2}{(e^x - e^{-x})^2}$$

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR
DOWNLOAD : GAMMA CAREER CLASSES AAP!

$$1-\frac{(e^x+e^{-x})^2}{(e^x-e^{-x})^2}$$

$$\frac{dy}{dx}=\boxed{1-\left(\frac{e^x+e^{-x}}{e^x-e^{-x}}\right)^2}$$

$$\boxed{\frac{dy}{dx}=1-y^2}$$ H.P

GAMMA CAREER CLASSES

DOWNLOAD GAMMA CAREER CLASSES AAP!

If $y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+\cdots\infty}}}$ Prove that $x(2y-1)\frac{dy}{dx}=1$

$$y=\sqrt{\log x+y}$$

Sq. Both.

$$y^2=\log x+y$$

$$2y\frac{dy}{dx}=\frac{1}{x}+\frac{dy}{dx}$$

$$2y\frac{dy}{dx}-\frac{dy}{dx}=\frac{1}{x}$$

$$\frac{dy}{dx}(2y-1)=\frac{1}{x}$$

$$\boxed{x(2y-1)\frac{dy}{dx}=1}$$ Ans

GAMMA CAREER CLASSES
MATHS BY ASHWANI SIR

DOWNLOAD : GAMMA CAREER CLASSES AAP!